Demonstrate understanding of wave systems

Subject Reference Physics 3.3

Title Demonstrate understanding of wave systems

Level 3 **Credits** 4 **Assessment** External

This achievement standard involves knowledge and understanding of phenomena, concepts, principles and/or relationships related to wave systems, and the use of appropriate methods to solve related problems.

Achievement	Achievement with Merit	Achievement with Excellence
• Identify or describe aspects of phenomena, concepts or principles. • Solve straightforward problems.	• Give descriptions or explanations in terms of phenomena, concepts, principles and/or relationships. • Solve problems.	• Give explanations that show clear understanding in terms of phenomena, concepts, principles and/or relationships. • Solve complex problems.

1 Assessment will be limited to a selection of the following:

Phenomena, concepts and principles of wave systems:

Interference (quantitative) of electromagnetic and sound waves, including multi-slit interference and diffraction gratings; standing waves in strings and pipes; harmonics and overtones; resonance; beats; Doppler Effect (stationary observer).

Relationships:

$$d\sin\theta = n\lambda \qquad n\lambda = \frac{dx}{L} \qquad f' = f\,\frac{v_w}{v_w \pm v_s} \qquad v = f\lambda \qquad f = \frac{1}{T}$$

2 Real life situations will be used wherever possible. Requisite information about the context used will be supplied.

3 Formulae applicable to this achievement standard will be supplied.

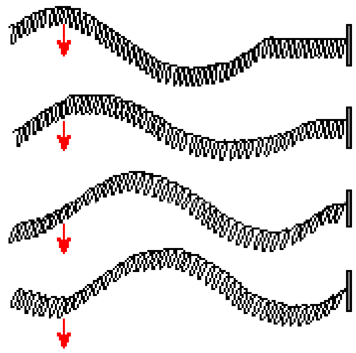
4 Students must be aware of the appropriate use of significant figures and units. Both negative index (e.g. m s^{-2}) and slash (e.g. m/s^2) notation will be acceptable when writing units. Negative index notation will be used when supplying data.

Wave Properties

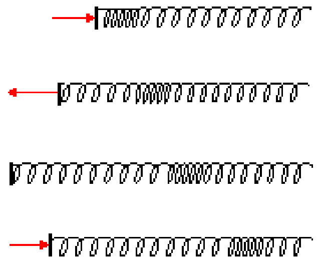
A wave is a regular vibration that carries energy.

Waves that travel through material media are of two basic kinds: **transverse** waves and **longitudinal** waves.

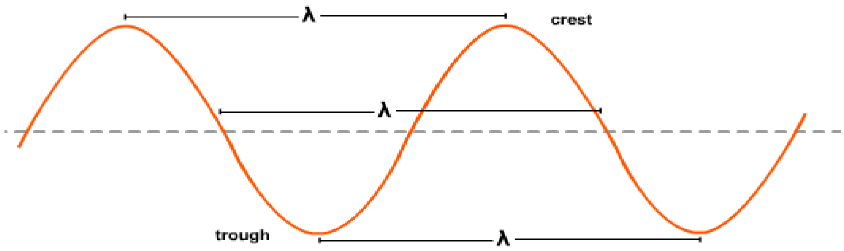
We can visualise both kinds of waves using a slinky spring stretched out on a smooth floor. Keeping one end fixed and shaking the other end from side to side will produce a **transverse** wave.



Keeping one end fixed and pushing the other end in and out will produce a **longitudinal** wave.



The frequency (f) of a wave is the number of oscillations per unit time. The frequency of a wave is the same as the frequency of the source that produces the wave. The unit of frequency is the **hertz**.



The period (*T*) of a wave is the time that it takes for one complete oscillation. The unit of period is the **second**. The period and frequency of a wave are linked by the following relationship,

$$f = \frac{1}{T}$$

The speed (*v*) of a wave is the distance travelled per unit time by the wave. The unit of speed is the **meter per second, ms^{-1}**. The speed, frequency and wavelength of a wave are linked by the following relationship,

$$v = f\lambda$$

The energy carried by a wave depends on the amplitude of the wave. The bigger the amplitude the more energy the wave carries.

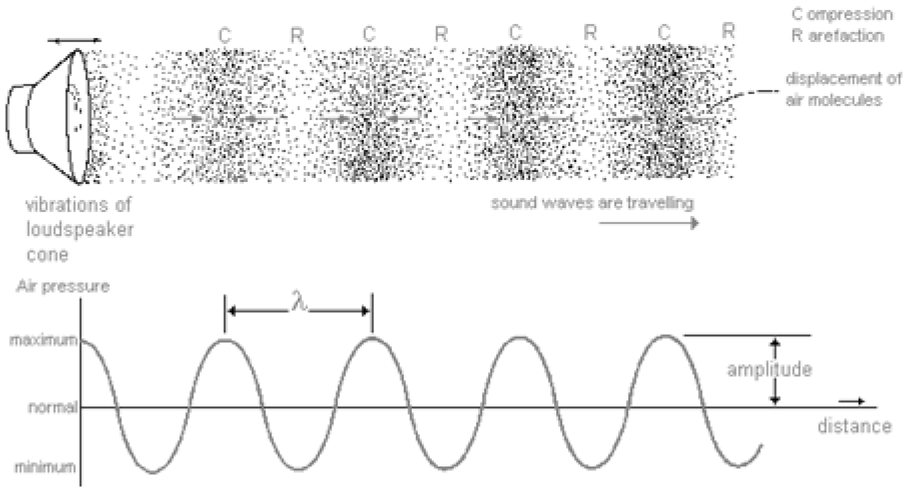
Electromagnetic Waves

Electromagnetic waves are transverse waves that travel at the speed of light in a vacuum.
Electromagnetic waves travel at different speeds in different media.

Radio	Low frequency	Remember
Microwaves	↓	My
Infrared		Instructions
Visible (ROYGBIV)		Visible
Ultra Violet		Under
X rays		X ray
Gamma rays	High frequency	Glasses

Sound waves

Sound waves are longitudinal waves consisting of a series of compressions and rarefactions.



The vibrations of the loudspeaker set up a series of compressions and rarefactions in the air. The graph of air pressure as a function of distance from the speaker is a sine curve.

Sound waves are longitudinal produced by vibrating objects which could be:

- a string which is plucked (guitar), bowed (violin) or hit (piano)
- a column of air in a wind instrument or organ pipe

The amplitude of the sound is the maximum displacement of the air particles from their rest position. It is also the difference between the maximum air pressure in a sound wave and normal air pressure. The greater the amplitude of the sound wave, the louder the sound.

Sound waves travel at different speeds in different media but a lot slower than electromagnetic waves.

material	temperature (°C)	speed (m s^{-1})
air	0	331
air	10	337
air	20	343
air	30	348
air	40	354
air	50	360
air	60	365
air	70	371
air	100	387
hydrogen	0	1286
water	20	410
glass	20	6000
brass	20	3500
steel	20	5000

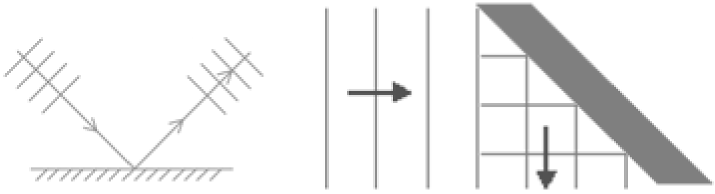
In air, sound travels at 331 ms^{-1} at a temperature of 0°C but the speed of sound changes with temperature. Sound travels faster in warm air than in cold air. The speed of sound in a medium depends on the density and compressibility of the medium so is different for different materials (much like light passing through different materials).

Wave Behaviour

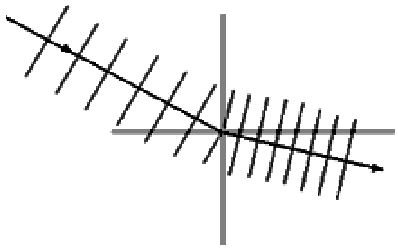
Reflection, refraction, diffraction and interference are behaviours of all types of wave.

Reflection occurs when a wave bounces from the surface of an obstacle.

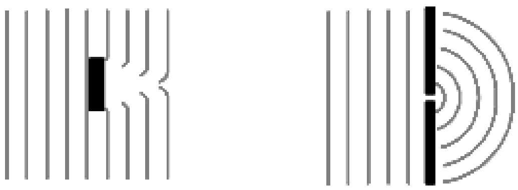
None of the properties of a wave are changed by reflection. The wavelength, frequency, period and speed are same before and after reflection. The only change is the direction in which the wave is travelling (and possibly a phase change causing it to invert).



Refraction occurs when a wave moves from one material to another. The speed and wavelength are changed by refraction. The frequency of the wave stays the same. The direction in which the wave is travelling may or may not be changed by refraction.



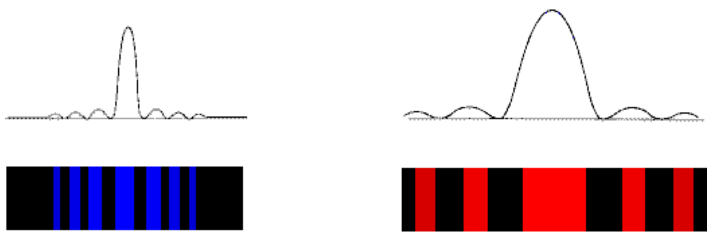
Diffraction occurs when a wave passes through around an object or through a gap (called a **slit** or an **aperture**).



When a wave passes through a gap the diffraction effect is greatest when the width of the gap is about the same size as the wavelength of the wave.

Smaller obstacles and smaller gaps lead to more diffraction or bending of waves than larger obstacles or gaps, when you are comparing waves with the same wavelength.

There is more diffraction or bending of waves with larger wavelength than of waves with smaller wavelength. The same happens with sound waves. This is the reason that you hear the thumping bass sounds from your local boy racers' car stereo without hearing the smaller-wavelength higher sounds of the melody. Diffraction of light using blue and red light:

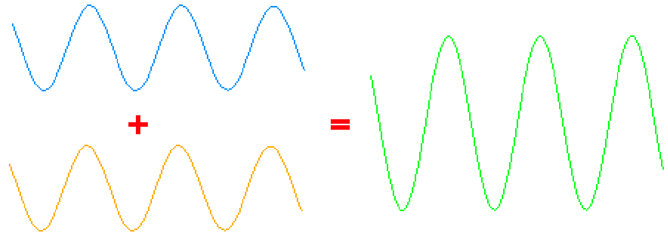


Diffraction is caused by interference between rays passing through a single slit. Notice that, unlike an interference pattern, the light bands in a diffraction pattern are of decreasing intensity and decreasing width as you look along from the center of the pattern.

The diffraction of light is harder to detect than the diffraction of sound because of the very small wavelength of light. Diffraction of sound waves through doorways and window openings is easily observed, and we can hear around corners, but we cannot see around corners.

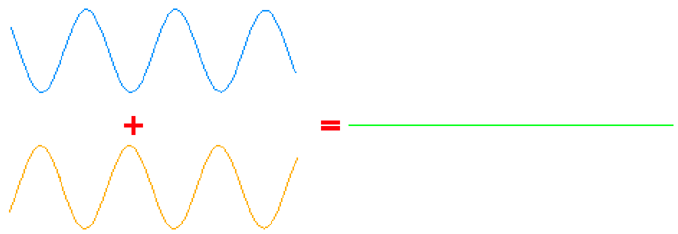
Interference

When waves run into each other, they usually don't reflect. Instead, they combine. If the amplitudes of two waves add up then the new wave has larger amplitude. This is called constructive interference.



When two waves of the same wavelength and frequency occur in the same place, they will have an effect on each other. If two waves are in sync, (the crest from one wave coincides with the crest from the other), they add up: this is a constructive interference.

If the waves had opposite amplitudes (one pointed up and the other pointed down), then the new wave has a smaller amplitude. This is called destructive interference.



If two waves are half a wavelength out of sync (the crests from one wave coincides with the troughs from the other), they cancel out, and the resulting wave will be zero; this is a destructive interference.

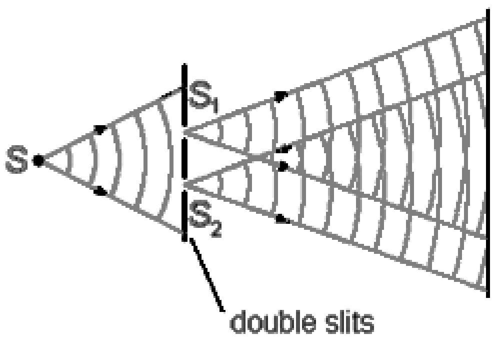
Constructive interference will make a sound louder while destructive interference will make a sound quieter.

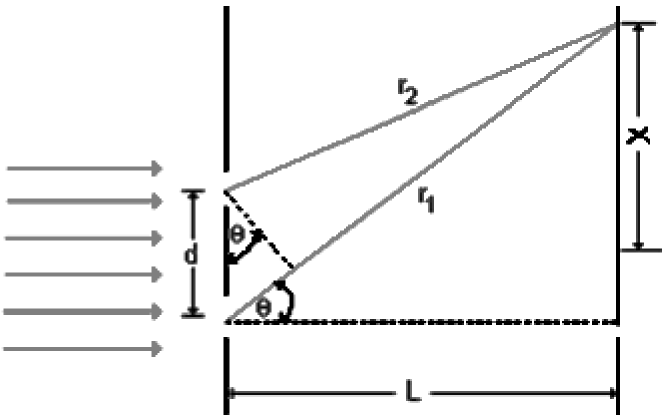
Two coherent sources of waves can produce an interference pattern. For coherence:

- The frequency of the sources is the same
- There is a constant phase relationship between the sources.

Young's Double-slit experiment

For any kind of wave, an interference pattern can be produced in a 'double-slit' experiment.



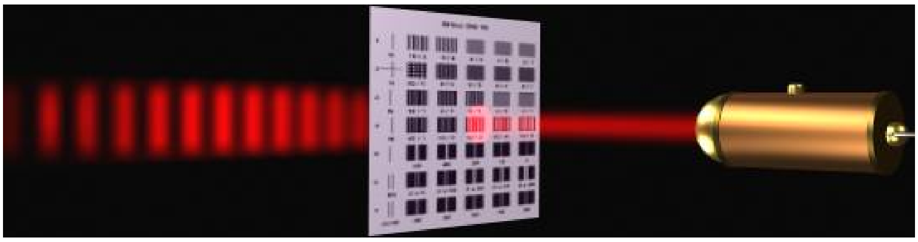


A wave detector may be moved across the interference pattern to find points of constructive and destructive interference. For visible light, a screen is used to show the interference pattern at the plane of the screen.

When **white light** is incident on a grating the central maximum is white. **Spectra** are produced at the other order maxima with blue light closest to the central maxima and red furthest. With a suitable grating, several **orders** of spectra may be observed.

The **central maximum** is also called the **zero order maximum**.

If **Monochromatic** light (light of one frequency) is used then **constructive interference** occurs at points where a wave from S_1 arrives in phase with a wave from S_2.



To model this effect mathematically, we begin with two slits separated at their mid-points by a distance *d*. The slits are very small compared to the wavelength of light. Two light rays, r_1 and r_2, originating from a single light source, pass through the slits and strike a screen at a distance L from the slits. A series of light and dark bands called fringes will be seen on the screen.

If the distance d << L, then the path difference between the two rays, $r_1 - r_2 = d \sin \theta$
Where:

θ is the angle between the light rays and a line drawn from a slit perpendicular to the screen.
x is the distance measured from a point on the screen opposite the center of the slits, and the point where the two rays meet. If $x << L$, then $\sin \theta \approx x/L$.

$$n\lambda = \frac{dx}{L}$$

Since the rays are in phase when they pass through the slits, constructive interference (bright bands of light) will occur when the path difference, $d \sin \theta$, is equal to a whole number of wavelengths.

$$d \sin\theta = n\lambda$$

For **constructive interference**, $d \sin \theta = n\lambda$ where n = 0, 1, 2, 3, ...

Destructive interference (dark bands) will occur when the path difference, $d \sin \theta$, is equal to an odd number of half-wavelengths.

Destructive interference occurs at points where a wave from S_1 and a wave from S_2 arrive out of phase.

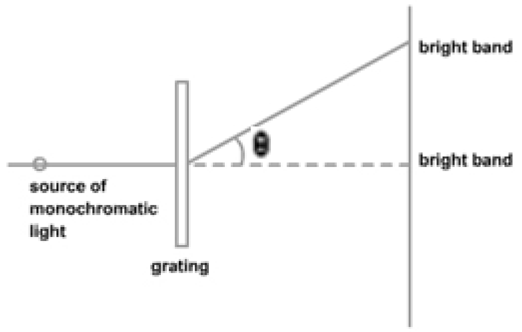
For destructive interference, $d \sin \theta = (n + ½) \lambda$ where n = 0, 1, 2, 3...

Gratings

A grating consists of many slits, or lines, close together. In a grating, the spacing between adjacent lines is constant. The distance between the slits, d = 1/n where n is the number of lines per metre.

Gratings are also used to produce interference patterns. When waves are incident on a grating all of the lines act as coherent sources of the waves.

Compared to a double slit, the interference pattern produced by a grating has fewer, more widely spaced points of maximum intensity.

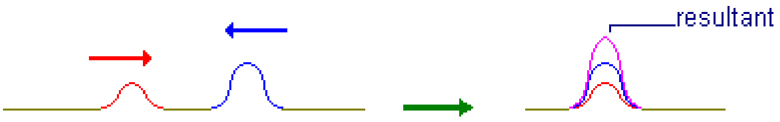


The diagram above illustrates the effect of a grating on a monochromatic source of light. For any particular grating *d* is constant and sin θ depends only on wavelength and so the bigger the wavelength the bigger the angle θ (hence why red is further from the central maxima).

Standing Waves

When two or more waves pass a point in a medium at the same time, their effects are added (algebraically) or superimposed.

Addition of waves: the combined effect of the waves is **constructive** if the waves are causing the particles to move in the same direction;



Addition of waves: the combined effect is **destructive** if the waves are causing the particles to move in opposite directions.

Resonance

Resonance depends on the fact that every object has its own natural frequency of vibration. If impulses are applied to an object with the same frequency as the natural frequency of vibration of the object, the object will begin to vibrate. If the applied impulses continue, the amplitude of vibration of the object will increase

Examples of resonance are:

*When pushing a child on a swing, the pushes are timed to coincide with the frequency of the swing, thus increasing the amplitude of the swing's vibration.

*A bridge has its own natural period of vibration. Soldiers marching over a bridge break step. This is because if their marching beat is the same as the bridge's vibration period, the bridge will be set into vibrations of increasing amplitude, which could eventually break the bridge.

*A piano string of a particular frequency can be set vibrating by a singer nearby singing a strong note of the same pitch.

*A singer can smash a nearby glass if she sings a note whose frequency is the same as the natural frequency of vibration of the glass.

Harmonics and overtones

When a pipe or a string is made to sound such as in a musical instrument the actual sound that is heard depends upon the mode of vibration. In the case of a pipe this depends upon how hard it is blown (blowing hard gives a higher note) and in the case of a string it depends upon the tension of the string the (greater the tension the higher the note).

In a pipe or string the lowest frequency is called the **fundamental**.

Overtones have a frequency that is an integral multiple of the fundamental and are said to form a **harmonic series**. The fundamental is the **first harmonic**.

If the fundamental (first harmonic) has a frequency **f** then the harmonic with a frequency of **2f** is called the **second harmonic** or **first overtone**

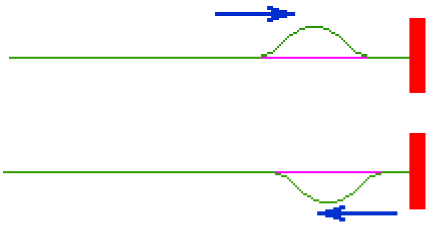
The harmonic with a frequency of **3f** is called the **third harmonic** or **second overtone**.

And so on and so on.

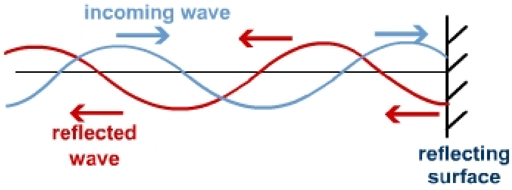
Each musical instrument adds ups these harmonics in a subtly different way. The particular addition of the harmonics gives the instrument its distinctive quality.

Standing waves: strings

Reflection and superposition can give rise to standing waves. The reflection of a pulse on a string with fixed end is shown below:



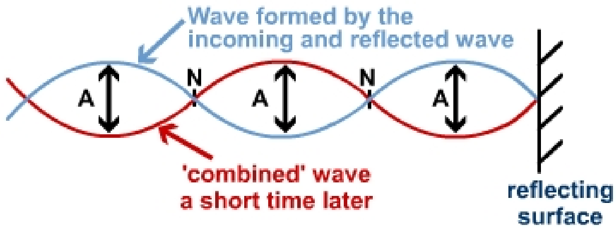
Strings fixed at each end are plucked in the middle; waves travel to each end and are reflected. The reflections cross and interfere to produce a standing wave with a frequency which is a natural or resonant frequency of the string. When a wave reflects, it comes back inverted (for example a crest becomes a trough).



The reflected wave and the incoming wave interfere. At the reflecting surface the two waves are always exactly equal and opposite - so they always cancel out. Such a place is called a **node(N)**. At other points along the waves, the two ways always are the same - so they add together or interfere constructively and make a double size wave. Such points are called **antinodes(A or AN)**.

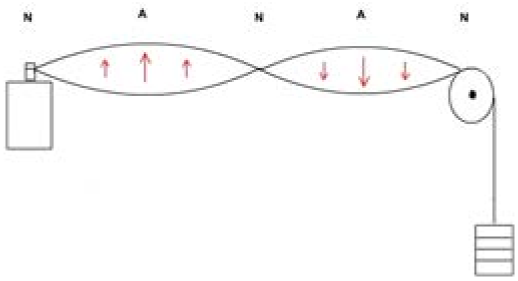
A - places where the waves interere constructively and make double height wave.

N - places where the two waves always 'cancel' out so there is no movement.

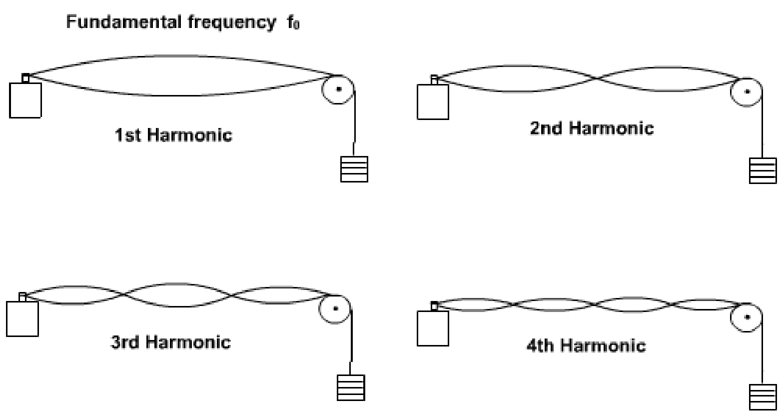


The distance between two **NODES** or between two **ANTINODES** is **half a wavelength, λ /2.**

The distance between a **NODE** and the next **ANTINODES** is **one quarter of a wavelength, λ /4.**

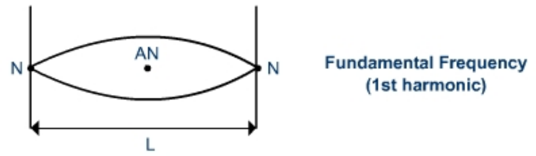


There is always a node at the fixed end of a vibrating string and there may be other nodes along the length of the string. The string may vibrate in several ways, known as harmonics.



Fundamental or first harmonic: L = λ/2; λ = 2L

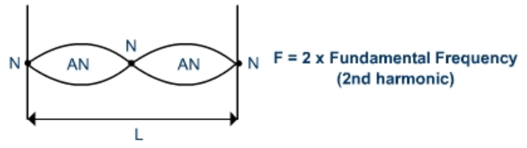
The lowest possible frequency standing wave that can fit on the string will be:



This is called the fundamental frequency, and it is the longest wavelength for that string.

Second harmonic: L = λ; λ = L

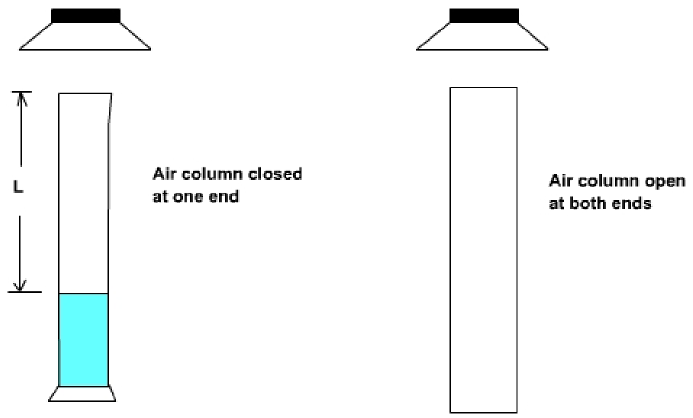
If we increase the frequency and decrease the wavelength, the next wave that will fit will be:



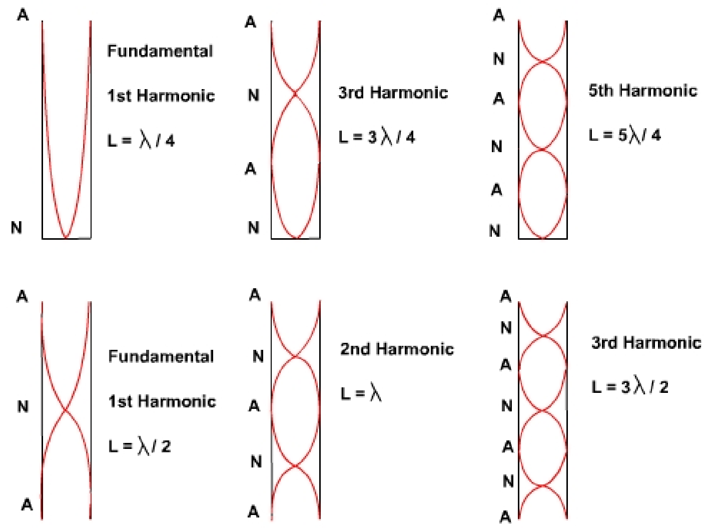
Third harmonic: L = 3 λ /2; λ = L / 3 etc…

Standing waves: pipes

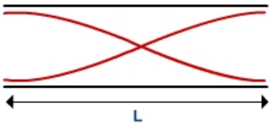
Pipes can be either blocked at one end or open at both ends.



A closed end of a tube must always be a node **and** an open end of a tube must always be an anti-node

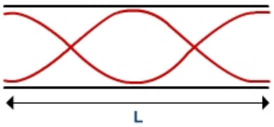


Open Pipes
So in the tube **open at both ends** the **fundamental wavelength** is 2L/1 where L is the length of the pipe.



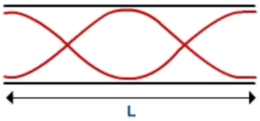
Wavelength = 2L
Frequency = f

The next possible oscillation mode is where wavelength = L. This is the **second harmonic or first overtone.**



Wavelength = 2L/2
Frequency = 2 f

The next possible oscillation mode is where wavelength = 2L/3. This is the **third harmonic or second overtone.**

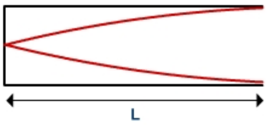


Wavelength = 3L/2
Frequency = 3 f

The next possible oscillation mode is where wavelength = 2L/4. This is **the fourth harmonic or third overtone,** etc.

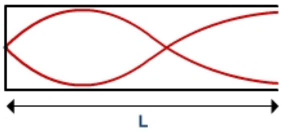
Closed Pipe

If the tube **open at one end,** the **fundamental wavelength** is 4/1 L (where L is the length of the pipe). The fundamental frequency standing wave that can fit in a pipe with one open end will be:



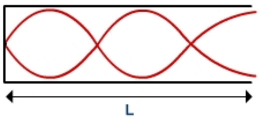
L = 1/4 x wavelength
Frequency = f

The **third harmonic or first overtone** is the next possible oscillation mode is where wavelength =4/3 L. Note: there cannot be a second (or 4^{th} or 6^{th} or…) harmonic because there is a node at one end and an antinode at the other.



L = 3/4 x wavelength
Frequency = 3 f

The next possible oscillation mode is where wavelength = 4/5 L. This is the **fifth harmonic or second overtone** etc.



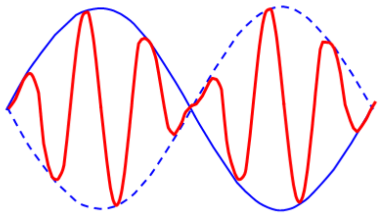
L = 5/4 x wavelength
Frequency = 5 f

End Correction: The waves don't end exactly at the open ends and instead will go slightly further out of the pipe. If you need to adjust for this in a calculation you just add on the extra distance to the length of the pipe.

Beats

When waves of the same type occupy the same point in space they will the overall effect is an algebraic summation of the waves. For this effect to be visible (in the case of light) or audible (in the case of sound) the waves must be of closely similar frequencies or the same frequency.

If the waves are similar but not the same, you will get beats as opposed to interference. This can be heard when you are tuning a guitar.



The closer the two frequencies the lower will be the beat frequency and this will become zero when they are perfectly in tune. The human ear is normally very sensitive to pitch and the two notes have to be close for beats to be heard.

Even if we take C and B on the musicians' scale, the beat frequency would be 262 - 247 = 15 Hz, about fifteen beats per second! This would not give audible beats, only an unpleasant discord.

Two waves that add together might have different frequencies. That means that the peaks won't always line up the same way because one wave is moving faster than the other. When this happens there are times when the waves interfere constructively and times when they interfere destructively.

It can be shown that the frequency of this variation is given by

$$|\mathbf{f_B}| = \mathbf{f_1} - \mathbf{f_2}$$

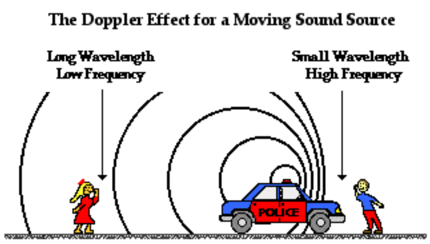
where $\mathbf{f_B}$ is the frequency of the beats and $\mathbf{f_1}$ and $\mathbf{f_2}$ are the frequencies of the two constituent waves e.g. a 440 Hz and 444 Hz wave combine to produce a beat of 4 Hz.

The frequency of the actual resultant wave is simply the average of the two constituent waves

The Doppler Effect

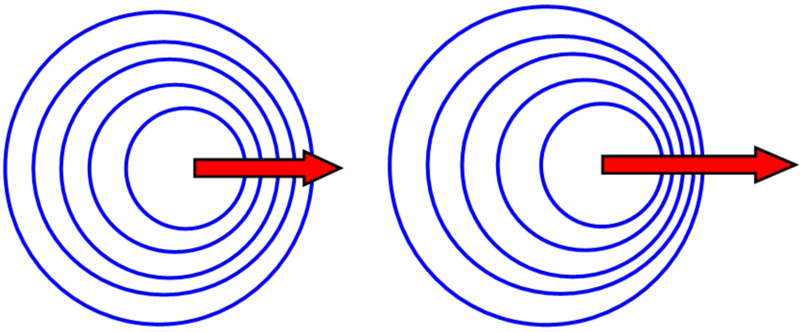
Beats are used in police radar speed traps. The outgoing and reflected signals are fed to the detector and the speed of the car is determined from the beat frequency using the **Doppler shift**.

When a car sounding its horn speeds past, the note heard by an observer changes pitch. When approaching, the pitch is higher; when moving away, the pitch is lower. This is an example of the Doppler effect: when there is relative motion between a source of sound and an observer, the observed (apparent) frequency is different from the actual frequency.



Moving source

In this case because the source is moving the waves are compressed ahead of the source and stretched out behind it. Thus an observer ahead of the source hears a higher apparent frequency, f ', and an observer behind the source hears a lower apparent frequency, f '. The greater the velocity of the source, the greater the Doppler effect.



$$f' = f \frac{v_w}{v_w \pm v_s}$$

Where f ' is the apparent frequency, f is the actual frequency, $\mathbf{v_w}$ is the velocity of the wave and $\mathbf{v_s}$ the velocity of the source.

You use:

- (v_w **-** v_s) if the motion is **towards** the observer
- (v_w **+** v_s) if the movement is **away** from the observer.

Proof of the equation for a stationary observer:

The one thing in all this that does not change is the actual speed of the waves through the medium (v_w) so

$$from \quad v = f\lambda \qquad so$$

$$we \quad get \quad v_w = f'\lambda' \qquad \frac{v_w}{f'} = \frac{v_w \pm v_s}{f_s}$$

$$\Rightarrow \lambda' = \frac{v_w}{f'} \qquad \Rightarrow f'(v_w \pm v_s) = f_s v_w$$

$$\Rightarrow f' = f_s \frac{v_w}{(v_w \pm v_s)}$$

Another use of the Doppler Effect is calculating the speed of stars due to the Doppler shift changing the relative position of the emission spectra from stars.