AS91524: Demonstrate understanding of mechanical systems

Level 3 Credits 6

Achievement	**Achievement with Merit**	**Achievement with Excellence**
Demonstrate understanding of mechanical systems.	Demonstrate in-depth understanding of mechanical systems.	Demonstrate comprehensive understanding of mechanical systems.

Assessment is limited to a selection from the following:

Translational Motion
Centre of mass (1 and 2 dimensions); conservation of momentum and impulse (2 dimensions only).

Circular Motion and Gravity
Velocity and acceleration of, and resultant force on, objects moving in a circle under the influence of 2 or more forces, Newton's Law of gravitation, satellite motion.

Rotating Systems
Rotational motion with constant angular acceleration; torque; rotational inertia; conservation of angular momentum; conservation of energy.

Oscillating Systems
The conditions for Simple Harmonic Motion, angular frequency, variation of displacement, velocity and acceleration with time, phasor diagrams, reference circles, damped and driven systems, resonance, conservation of energy.

Relationships

$d = r\theta$ $\quad v = r\omega$ $\quad a = r\alpha$ $\quad \omega = \frac{\Delta\theta}{\Delta t}$

$\alpha = \frac{\Delta\omega}{\Delta t}$ $\quad \omega = 2\pi f$ $\quad E_{K(ROT)} = \frac{1}{2}I\omega^2$

$\omega_f = \omega_i + \alpha t$ $\quad \theta = \frac{(\omega_i + \omega_f)}{2}t$ $\quad \omega_f^{\ 2} = \omega_i^{\ 2} + 2\alpha\theta$ $\quad \theta = \omega_i t + \frac{1}{2}\alpha t^2$

$\tau = I\alpha$ $\quad L = mvr$ $\quad L = I\omega$ $\quad F_g = \frac{GMm}{r^2}$

$T = 2\pi\sqrt{\frac{l}{g}}$ $\quad T = 2\pi\sqrt{\frac{m}{k}}$

$y = A\sin\omega t$ $\quad v = A\omega\cos\omega t$ $\quad a = -A\omega^2\sin\omega t$ $\quad a = -\omega^2 y$

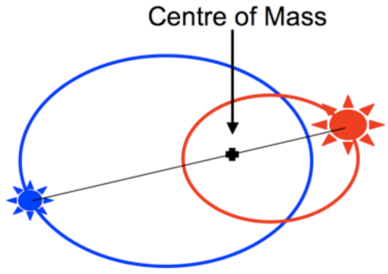
$y = A\cos\omega t$ $\quad v = -A\omega\sin\omega t$ $\quad a = -A\omega^2\cos\omega t$

$$x_{COM} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

This achievement standard replaced unit standard 6397 and AS90521.

Centre of Mass

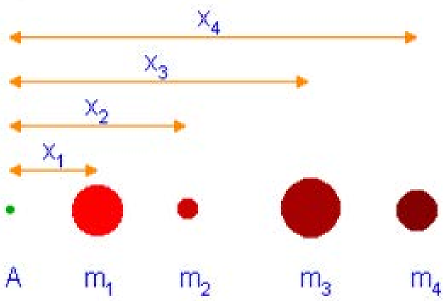
The centre of mass is the point at which all the mass of an object appears to be concentrated as far as the object's motion is concerned. In simple terms it is the balance point of an object, the place where an object must be supported if it is to remain balanced.



Two stars that are gravitationally bound are said to orbit their common centre of mass.
When an external force acts upon a system of particles, the particles react as though all the matter were concentrated at the centre of mass in accord with Newton's second law of motion. Thus if we understand how the centre of mass is moving, we can understand the net force on a system. In a system with many parts the position of the centre of mass from any point can be found using the following formula:

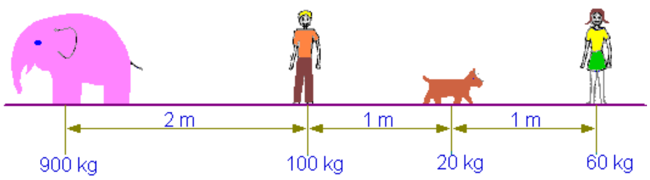
$$X_{cm} = \frac{x_1m_1 + x_2m_2 + x_3m_3 + x_4m_4 + ..}{m_1 + m_2 + m_3 + m_4 + ..}$$

where
X_{cm} = distance from refenence point (A) to centre of mass
m_1 = mass of object 1
x_1 = distance from reference point (A) to object 1



Note that the distances (x_1, x_2, x_3,..) are measured from any arbitrarily chosen origin point. It simplifies calculation if the origin is chosen to coincide with one of the masses.

How far from the centre of mass of the elephant is the centre of mass of the four body system?



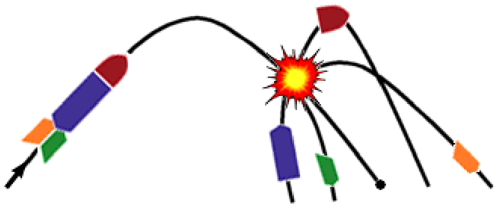
Using the centre of mass of the elephant as the reference point:

$$X = \frac{(900 \times 0) + (100 \times 2) + (20 \times 3) + (60 \times 4)}{(900 + 100 + 20 + 60)}$$

X = 0.46 m from elephant

Centre of mass during collisions

Consider a missile composed of four parts, traveling in a parabolic path through the air. At a certain point, an explosive mechanism on the missile breaks it into its four parts, all of which shoot off in various directions.



The only external force that acts upon the system is gravity, and it acts in the same way it did before the explosion. So even though the missile pieces fly off in unpredictable directions, the center of mass of the four pieces will continue in the same parabolic path it had travelled in before the explosion.

The centre of mass remains at rest, or travelling at a constant velocity unless an **external force** acts on the system.

Centre of mass

The centre of mass (or centre of gravity) of a body is the point through which all the mass appears to act. For a symmetrical body the CM is in the centre. The centre of mass does not have to be contained within the body. For a body to be stable the centre of mass must lie either below the point of suspension, or above the base area.

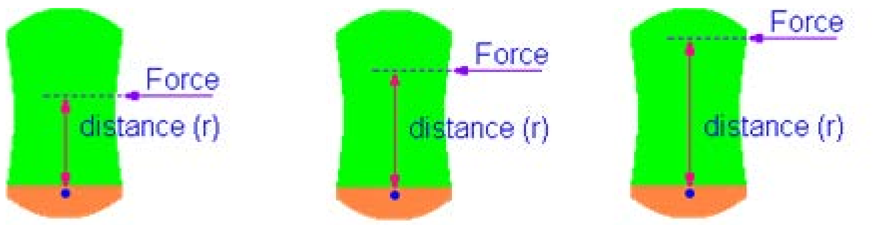


Figure 3 Torque about a given point
= force × perpendicular distance from the point to the line of action on the force

Any external force applied to the body which is not in line with the centre of mass will result in a rotation of the body (Fig (a)). If the vertical line from the centre of mass falls outside the supporting base, then the object is unstable and will rotate about the point of contact (falls over), see Fig (b). To make this object more stable either increase the area of its base or lower the centre of mass (Fig. (c)).

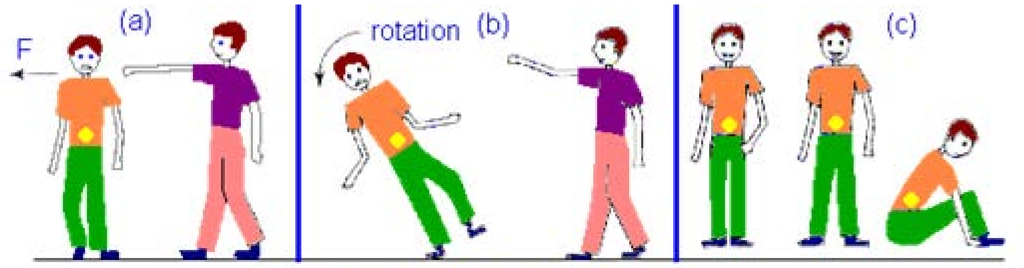


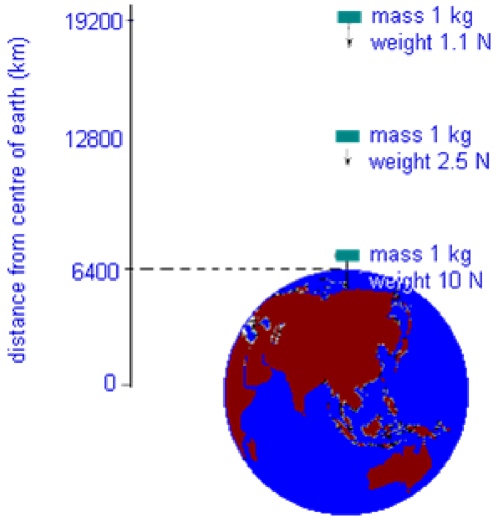
Figure 4 Rotational stability of a body:
(a) rotation caused by an external force
(b) rotation of an unstable body
(c) increasing the stability of a body

Forces

Gravitational Forces

A downward force is produced on all masses near the earth's surface by our planet's gravitational attraction. This downward force on an object is called the object's **weight**.

At the earth's surface, the gravitational field strength (g) is about 9.81 N kg^{-1}, so an object with a mass of 60 kg, such as a person, has a weight of 587 N. Mass is the 'quantity of matter' in an object, and is a measure of how hard it is to accelerate the object. The **mass** of an object is the same on earth, on the moon or in outer space.



Weight is the force exerted on a mass by other masses. The weight of an object depends on what other masses are around. The weight of an object is different on earth, on the moon or in outer space.

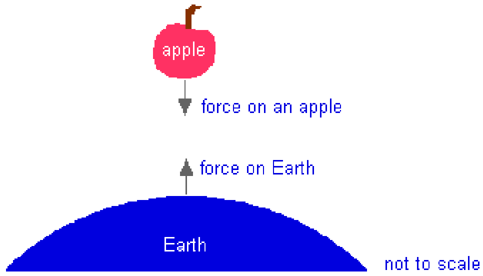
$$F_g = mg$$

where

F_g = weight (N)

m = mass (kg)

g = gravitational field strength (N kg^{-1})



Every mass attracts every other mass. We don't notice it for most masses because it is tiny in size, due to the value of G, the Universal Constant of Gravitation:

$$G = 6.67 \times 10^{-11}\ \text{N m}^2\ \text{kg}^{-2}$$

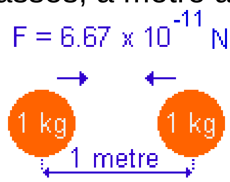
Newton provided an equation for the force of gravity:

$$F_g = \frac{G\, M_1\, M_2}{d^2}$$

where

F_g	= force of attraction between 2 masses	(N)
G	= the universal gravitational constant	($\text{N m}^2\ \text{kg}^{-2}$)
M_1	= one of the masses	(kg)
M_2	= the other mass	(kg)
d	= distance between centres of mass of the two objects	(m)

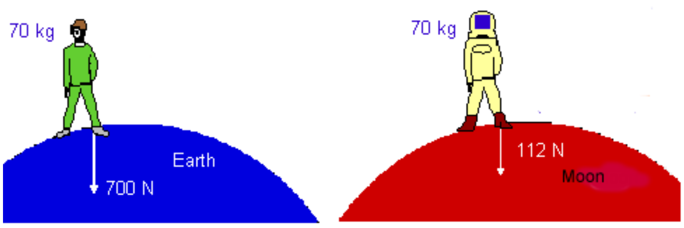
This is not noticeable between 2 one kg masses, a metre apart.



It is only when at least one of the masses involved in the attraction is large that the force becomes noticeable! The earth has a mass of 5.98×10^{24} kg and so exerts a noticeable force even on small

G is a universal constant as it has the same value everywhere in the universe

g, [the gravitational field strength]() at the earth's surface, is an approximate local constant; approximate and local because it varies a little from place to place about the earth and has the value of 9.8 N kg^{-1} only in the locality of the earth's surface. On the moon, the local gravitational field strength is about 1.6 N kg^{-1}.



Satellites

To launch a satellite in space, multi-stage rockets are used. They raise the satellite to a predetermined height and then project it in the right direction with a velocity which enables it to revolve around the Earth. This velocity is called the escape velocity. Escape velocity is the velocity with which a body should be projected to enable it to escape from the gravitational influence of the Earth.

$$F = \frac{GMm}{R^2}$$

If v_e is the escape velocity then

$$\frac{1}{2}mV_e^2 = \frac{GMm}{R} \quad V_e^2 = \frac{2GM}{R} \quad V_e = \sqrt{\frac{2GM}{R}} \quad \left[g = \frac{GM}{R^2}, \therefore GM = gR^2\right]$$

$$V_e = \sqrt{\frac{2gR^2}{R}} = \sqrt{2gR}$$

If $g = 9.8 ms^{-2}$ and $R = 6.4 \times 10^6 m$ Then $v_e = 11.2 \times 10^3 ms^{-1}$

Orbital velocity

The velocity with which, an object circles the Earth in a circular path is known as orbital velocity. The centripetal force required make the satellite go around in a circular orbit acts towards the centre of the Earth and this force is made by the gravitational force of attraction

$$F = \frac{GMm}{R^2} \therefore \frac{mv^2}{R} = \frac{GMm}{R^2}$$

$$V^2 = \frac{GM}{R}$$

which also acts towards the centre of the Earth. $\left[g = \frac{Gm}{R^2}, \therefore Gm = gR^2\right]$

Substituting for Gm in the equation, we get $V = \sqrt{\frac{gR^2}{R}} = \sqrt{gR}$

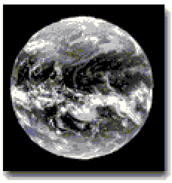
$\therefore$The orbital velocity can be calculated taking $g = 9.8 ms^{-2}$ and $R = 6.4 \times 10^6 m$
$\therefore v = 7.9 \times 10^3 ms^{-1}$

Uses of artificial satellites

They can be used for

- weather forecasting
- studying the atmosphere
- telecommunication
- mapping the Earth

Geostationary satellite

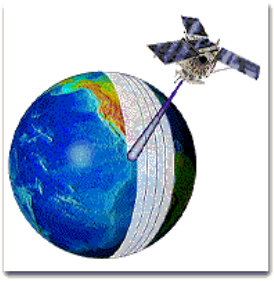


A satellite, whose period of revolution is 24 hours, is a geostationary satellite. It always appears to be at a fixed point in space, because the period of rotation of the Earth about its own axis is also equal to 24 hours. Knowing T = 24 hours, g = 9.8 ms^{-2}, the height of a geostationary satellite is calculated to be 36000km.

- Its orbital velocity is 3.1 kms^{-1}
- Its plane of orbit is the equatorial plane.
- It revolves from west to east which is similar to the Earth's movement.
- It is very useful in telecommunication.

In order for the satellite to be geostationary, the orbit must also be in the plane of the equator. Imagine a satellite in orbit 36,000 kilometres above the ground, but moving in a circle that took it over the north and south poles. This satellite would take 24 hours to complete an orbit, but it would not be geostationary because it would rotate in a different direction than the earth (north-south instead of east-west) and so wouldn't remain above the same point on the ground.

Polar satellites

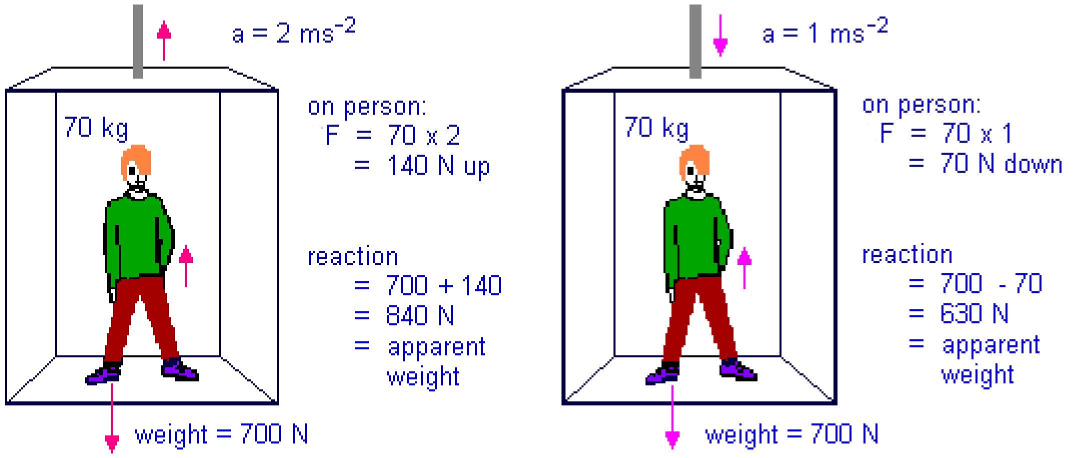


Satellites in polar orbits are used for environmental and earth resources' survey. They move over the Polar Regions and cover the whole of the Earth's surface in a few weeks.

Apparent Weight

Astronauts can appear weightless even while in a strong gravitational field. The gravitational field at the Space Station 250 km above the Earth's surface is 9.2 Nkg^{-1}.

Weight is the force exerted on a mass by the gravitational field it is in. Apparent weight is the reaction force on the mass from the surface supporting it.
Apparent weight can be different from weight. When a lift starts or stops the passenger is accelerated vertically. There is a net force- the apparent weight. While weight stays the same during acceleration (because gravity doesn't change!) the reaction force does change. The passengers experience the reaction force as apparent weight and may feel heavier or lighter as the lift starts or stops.



Weight and apparent weight in a lift

Passengers can experience weightlessness in an aircraft which flies in a vertical loop so that its downward acceleration at the top of the loop is 9.8 ms^{-2}. This is the acceleration of an object which is falling. The aircraft, being accelerated downward at 9.8 ms^{-2} is effectively falling. If net force $=mv^2 / R = mg$ then reaction force = 0 so apparent weight is zero - "weightlessness".

Momentum

Moving objects are hard to stop. This can be understood using Newton's second law of motion. The mass × velocity product is very useful and it is given the name momentum. Momentum is a vector and is given the symbol p. It is measured in kg m s^{-1}.

Moving objects resist being stopped, a quality which we call their inertia.

We experience a force when struck by a moving object.
Change in momentum, $\Delta p = F\,t$

The force × time product is called impulse. Impulse is force multiplied by the time for which it acts. Impulse is a vector. It is not usually given its own symbol. It is measured in Ns.

Since impulse = change in momentum, the units kg m s^{-1} and Ns are equivalent.
Momentum is conserved in a collision. The force between the colliding objects is a force of interaction, one that the objects exert on one another.

The total momentum at the time the objects start to interact is equal to the total momentum at the time their interaction finishes.
The total can stay constant because momentum is a vector. Whether the collision is elastic or inelastic makes no difference to the conservation of momentum.

Kinetic energy

Energy is conserved during a collision (it always is!) but kinetic energy is only conserved in some types of collisions.
Kinetic energy remains constant in what is called an elastic collision.
Kinetic energy is converted to other types of energy (e.g. heat, sound) in what is called an inelastic collision.

Forces during a collision

When a car collides its momentum changes. So does the momentum of its occupants.
There is a force connected with this momentum change:

$$\vec{F}\Delta t = \overrightarrow{\Delta p}$$

$$\vec{F} = \frac{\overrightarrow{\Delta p}}{\Delta t}$$

The bigger the momentum change, the greater the force.

Bouncing off something during a collision can lead to up to double the momentum change and so up to double the force.

The shorter the time of the collision, the greater the force.

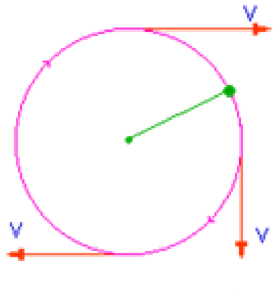
To minimise injury the force must be kept as small as possible. The momentum change for a passenger depends on initial and final speed (and their mass!). Force can be reduced if the time for the collision is increased - have the momentum change take place over a long time rather than a short one. Seat belts (they stretch a little), air bags and crumple zones on cars all have the effect of increasing collision time and so reducing the force on the passenger.

In all interactions there are equal but opposite forces on each object (Newton’s third law) and as the time the forces act must be the same on both, this means that there must be equal but opposite momentum changes ($\Delta p = F\ t$) on each object. The total momentum in the interaction remains constant. In general, the total momentum of a collection of objects remains constant when the only forces are those the objects exert on one another. Consider an apple held above the ground. There is a force downward on it from the earth’s gravity. If it is released it falls. It gains velocity. Its momentum increases. Where is conservation of momentum in this simple situation?

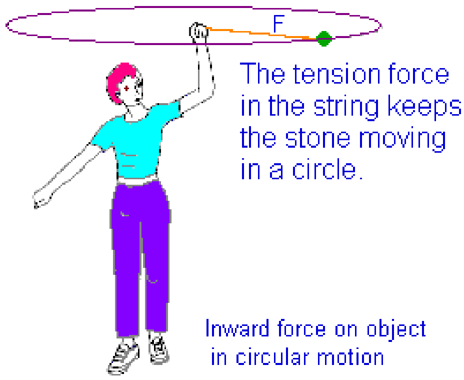
Circular motion

An object moving in a circle has its *direction is changing all the time.*

This means that even if the object's *speed* is constant its *velocity* isn't.



Changing velocity means acceleration and for acceleration to occur there must be a net force. Every object moving in a circular path is accelerating. Every object moving in a circular path must have a net or unbalanced force on it to produce that acceleration.



The direction of the force is inward towards the centre of the circle.

For something moving with constant speed:
* Motion once around a circular path is called a revolution
* The time for one revolution is called a period (T)

The number of revolutions per second is called the frequency, f

$$T = \frac{1}{f} \quad \text{or} \quad f = \frac{1}{T}$$

where

T = seconds per revolution
f = revolutions per second

* Distance travelled in one revolution = circumference of circle = $2\pi R$
  where
  R = radius of circle (m)

* Speed = $v = \dfrac{\text{distance}}{\text{time}}$

$$v = \frac{2\pi R}{T}$$

where
v = speed (m s^{-1})
R = radius (m)
T = period (s)

alternatively $\quad v = 2\pi R f$

where
f = frequency (Hz)

Centripetal force can be calculated by:

$$\text{centripetal force} = \frac{m v^2}{R}$$

where

m = mass (kg)

v = speed (m s^{-1})

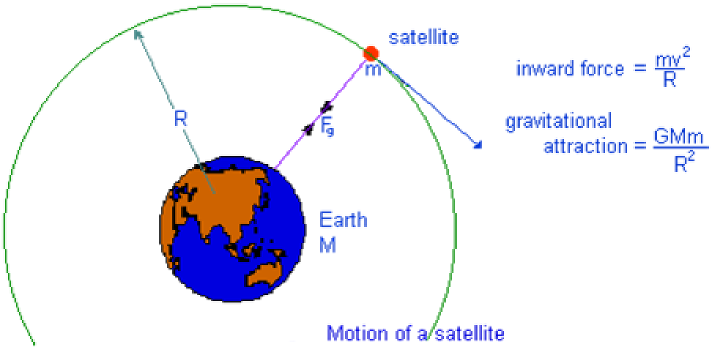
R = radius (m)

Note: The centripetal force that makes an object undergo circular motion is a result of other forces.

Specific examples of Circular motion

1. Satellites

The velocity with which, an object circles the Earth in a circular path is known as orbital velocity.



The centripetal force required make the satellite go around in a circular orbit acts towards the centre of the Earth and this force is made by the gravitational force of attraction

$$F_c = F_g$$

$$\frac{4\pi^2 m R}{T^2} = \frac{G M m}{R^2}$$

where

F_c = centripetal force on satellite

F_g = gravitational force on satellite

R = radius of orbit

T = period of motion

G = universal gravitational constant

M = central mass

m = orbiting mass

rearranging gives:

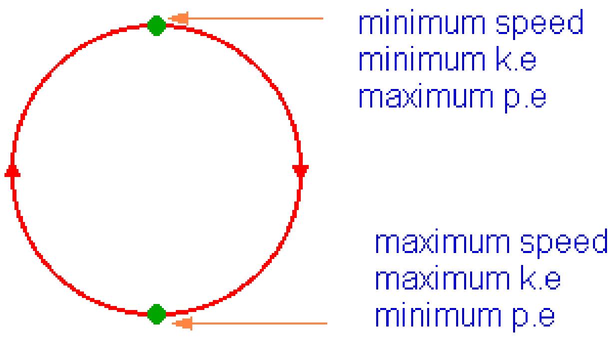
$$T^2 = \frac{4\pi^2 R^3}{G M}$$

$$T = \sqrt{\frac{4\pi^2 R^3}{G M}}$$

This is an example of a single force providing the centripetal force.

2. Vertical circles

Where an object near the earth's surface is moved around in a vertical circle, its speed will alter as gravitational potential energy at the top point of the motion is converted to kinetic energy at the bottom.

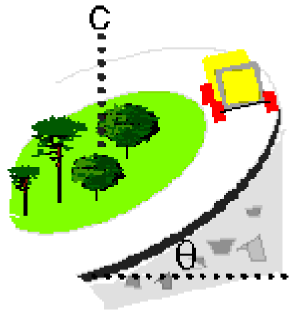


Energy changes in vertical circular motion

For such an object there are at least two forces affecting its motion: the inward applied force required to accelerate an object in a circular path (usually Tension if it is a structure) and the weight force.

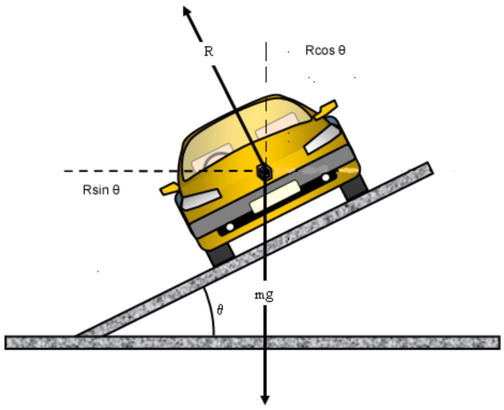
3. Banked Tracks

A car on a banked curve will experience a centripetal force if it is moving in an arc (part of a circle).

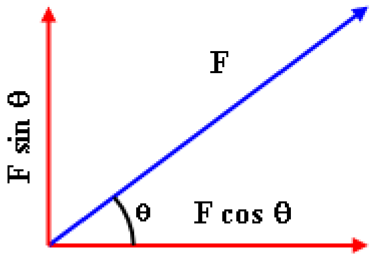


car on banked curve with centre C

Looking at it "end-on", here is the free-body diagram (FBD):



The Reaction force can be divided into a vertical and horizontal component in the same way any vector can:



$R \sin\theta$ is the centripetal force directed towards the centre of the curve (not parallel with the bank!)

Since $R \sin\theta$ supplies the centripetal force,

$$R \sin\theta = \frac{mv^2}{r}$$

$$R \cos\theta = mg$$

$$\frac{R \sin\theta}{R \cos\theta} = \frac{mv^2/r}{mg}$$

$$\tan\theta = \frac{v^2}{rg}$$

$$r = \frac{v^2}{g \tan\theta}$$

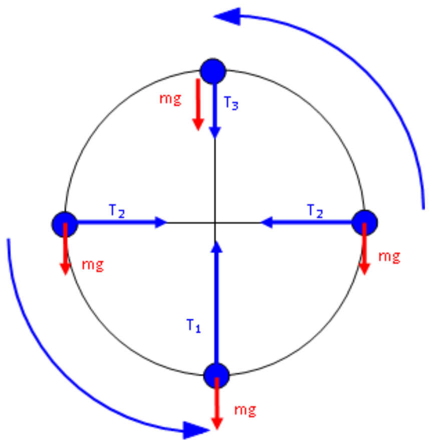
$$v = \sqrt{rg \tan\theta}$$

If the car moves faster than this speed, it will slip up the banked curve, and if it goes slower, it will slip down the bank (assuming the road is perfectly frictionless).
Heavy cars and light cars behave identically on banked curves.

This same Physics theory can be applied to conical pendulums and aircraft banking during a turn.

Motion in a vertical circle

If an object is being swung round on a string in a vertical circle at a constant speed the centripetal force must be constant but because its weight (mg) provides part of the centripetal force as it goes round the tension in the string will vary.



Let the tension in the string be T_1 at the bottom of the circle, T_2 at the sides and T_3 at the top.

At the bottom of the circle: $T_1 - mg = mv^2/r$ so $T_1 = mv^2/r + mg$

At the sides of the circle: $T_2 = mv^2/r$

At the top of the circle: $T_3 + mg = mv^2/r$ so $T_3 = mv^2/r - mg$

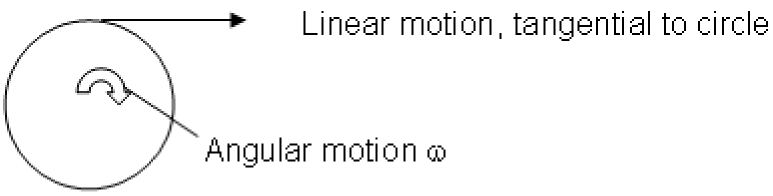
So, as the object goes round the circle the tension in the string varies being greatest at the bottom of the circle and least at the top. Therefore if the string is to break it will be at the bottom of the path where it has to not only support the object but also pull it up out of it straight-line path.

Comparing linear and angular motion

	Linear Motion	Rotational Motion	
Position	d	θ	Angular position
Velocity	v	ω	Angular velocity
Acceleration	a	α	Angular acceleration
Motion equations			Motion equations
	$v_f = v_i + at$	$\omega_f = \omega_i + \alpha t$	
	$d = v_i t + \frac{1}{2}at^2$	$\theta = \omega_i t + \frac{1}{2}\alpha t^2$	
	$v_f^2 = v_i^2 + 2ad$	$\omega_f^2 = \omega_i^2 + 2\alpha\theta$	
Mass (linear inertia)	m	I	Moment of inertia
Newton's second law	$F = ma$	$\tau = I\alpha$	Newton's second law
Momentum	$p = mv$	$L = I\omega$	Angular momentum
Work	Fd	$\tau\theta$	Work
Kinetic energy	$\frac{1}{2}mv^2$	$\frac{1}{2}I\omega^2$	Kinetic energy

Basic Rotational Quantities

Rotation is described in terms of angular displacement, time, angular velocity, and angular acceleration.



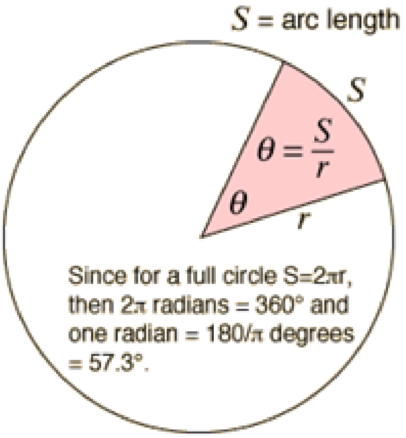
Remember:

2π radians in a circle or revolution, means that $1 rpm = \frac{2\pi}{60} rads^{-1}$

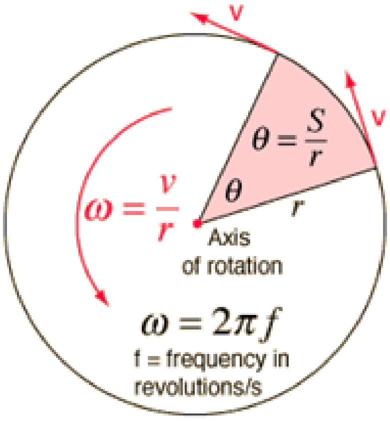
Multiply angular quantity by radius to convert to tangential linear quantity.

Angular displacement:

The standard angle of a directed quantity is taken to be counterclockwise from the positive x axis.



Angular Velocity:



Angular velocity can be considered to be a vector quantity, with direction along the axis of rotation in the right-hand rule sense.
For an object rotating about an axis, every point on the object has the same angular velocity. The tangential velocity of any point is proportional to its distance from the axis of rotation. Angular velocity has the unit rads^{-1}

$$v = \omega r \quad or \quad \omega = \frac{v}{r}$$

Angular velocity is the rate of change of angular displacement and can be described by the relationship

$$\omega_{average} = \frac{\Delta\theta}{\Delta t}$$

Angular Acceleration:

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

The equations of motion met at NCEA Level 2 can be used to solve rotational problems.

$$\omega_f = \omega_i + \alpha t$$

$$\theta = \frac{(\omega_i + \omega_f)}{2} t$$

$$\omega_f^{\,2} = \omega_i^{\,2} + 2\alpha\theta$$

$$\theta = \omega_i t + \tfrac{1}{2}\alpha t^2$$

Rotational Inertia (I)

The rotational inertia describes how the mass is arranged around the center of the rotation.

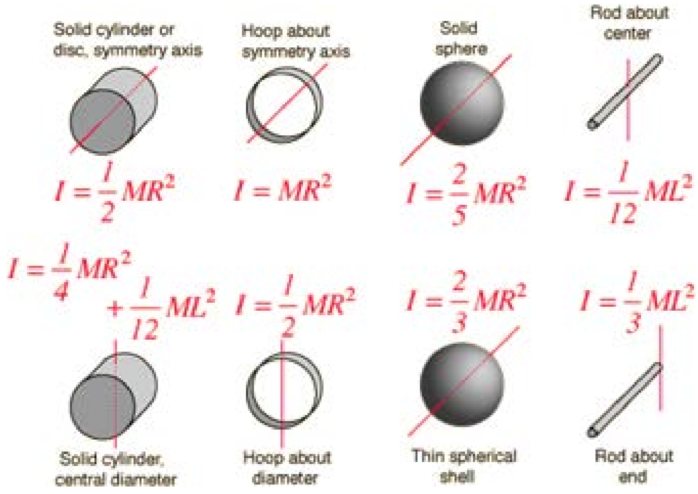
The moment of inertia is I = Σ mr^2.

The moment of inertia – as the equivalent of mass in linear equations – is fundamental to a number of equations.

Moment of Inertia I

Quantity	Linear	Angular
Newton's Second Law	$F = ma$	$\tau = I\alpha$
Kinetic Energy	$KE = \frac{1}{2}mv^2$	$KE = \frac{1}{2}I\omega^2$
Momentum	$p = mv$	$L = I\omega$
Work-Energy	$F_{net}d = \Delta\left(\frac{1}{2}mv^2\right)$	$\tau_{net}\theta = \Delta\left(\frac{1}{2}I\omega^2\right)$

The moment of inertia is different for different shapes.



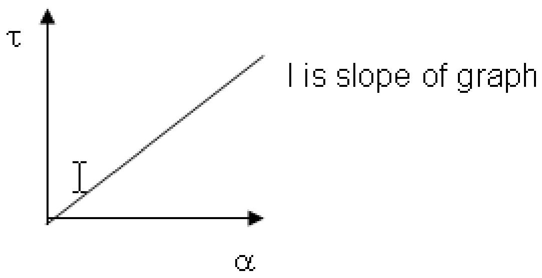
Torque or turning force, causes angular acceleration.

If we want to make a wheel rotate we exert a tangential force on its rim. The turning effect of a force or torque or moment of a force:

$$\tau = F\,d$$

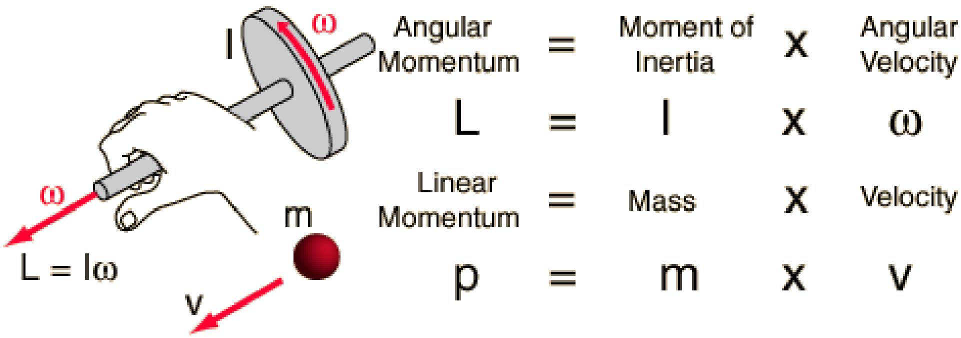
The unit of torque is the newton metre (N m)

$$\tau = I\alpha$$



Angular Momentum

The angular momentum of a rigid object is defined as the product of the moment of inertia (I) and the angular velocity (ω). Similar to linear momentum - angular momentum is conserved if there is no external torque on the object. Angular momentum is a vector quantity. (Unit is kg^2s^{-1})

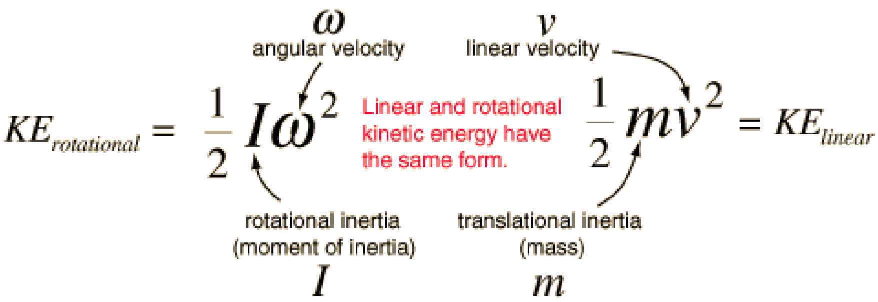


Note $L=mvr_{\perp}$ - links linear momentum with angular momentum.

Rotational Kinetic Energy

The kinetic energy of a rotating object is analogous to linear kinetic

$$KE_{rotational} = \frac{1}{2} I\omega^2$$



Rotational kinetic energy can be stored in flywheel e.g. toy cars

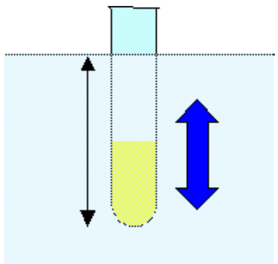
Since E_k depends on I a hollow cylinder will have greater Rotational E_K than a solid cylinder.

This means that a solid cylinder of same mass will roll down a slope faster than a hollow cylinder since more gravitational E_P is converted to Rotational E_K (for the hollow cylinder).

Simple harmonic motion

Any motion that repeats itself after a certain period is known as a **periodic** motion, and since such a motion can be represented in terms of sines and cosines it is called a **harmonic** motion.

Simple harmonic motion (S.H.M. for short) is the name given to a particular type of harmonic vibration. The following are examples of simple harmonic motion:



- a test-tube bobbing up and down in water
- a simple pendulum
- a compound pendulum
- a vibrating spring
- a vibrating cantilever
- a marble on a concave surface
- liquid oscillating in a U-tube

Simple harmonic motion is defined as follows:

A body is undergoing simple harmonic motion if it has an acceleration which is:
(a) directed towards a fixed point, and
(b) proportional to the displacement of the body from that point.

SHM requires the same magnitude of force irrespective of direction of displacement – which is why bouncing on a trampoline, is NOT SHM.

The equation for the acceleration of a body undergoing simple harmonic motion is usually written as (where y is measured from the equilibrium position):

$$a = -\omega^2 y$$

The solution to this equation can be shown to be of the form:

$$y = A \sin \omega t$$

Since the equation for acceleration can be obtained by twice differentiating the equation for displacement against time.

If we differentiate this equation we have equations for v:

$$v = A\omega \cos \omega t$$

There are alternative equations for SHM based on measurements from the extremities of the swing:

$$y = A \cos \omega t$$

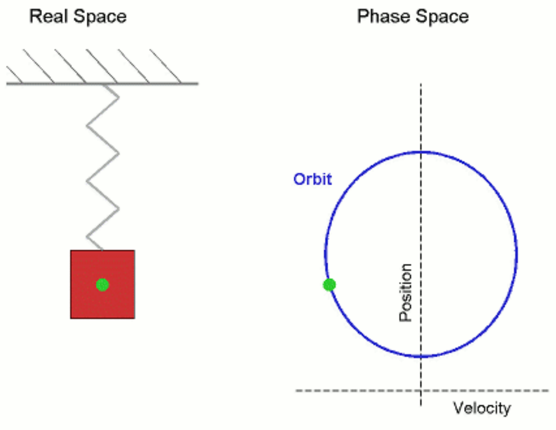
$$v = -A\omega \sin \omega t$$

$$a = -A\omega^2 \cos \omega t$$

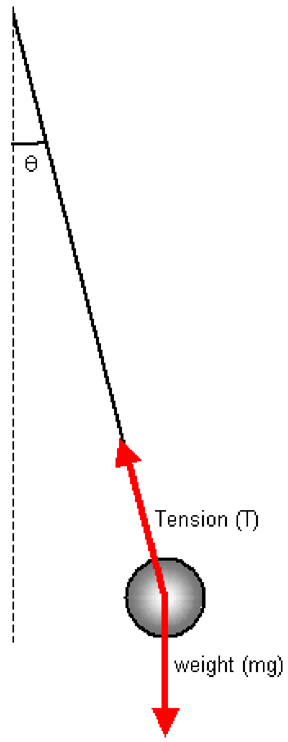
The time period of an oscillation for SHM is given by:

Period (T) = $2\pi/\omega$

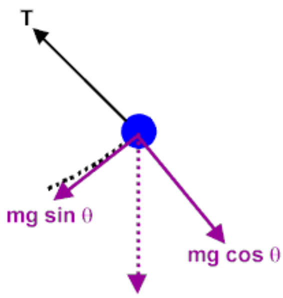
There is a relationship between angular mechanics and SHM which can be represented as below:



The simple pendulum



As the pendulum swings, it is accelerating both centripetally, towards the point of suspension and tangentially, towards its equilibrium position. It is its linear, tangential acceleration that connects a pendulum with simple harmonic motion.



The weight component, mg sin θ, is accelerating the mass towards equilibrium along the arc of the circle. This component is called the restoring force of the pendulum.

The restoring force F is the component of the weight of the bob. Therefore

$F = - mg \sin \theta = ma$ giving $a = - g \sin \theta$

But for small angles $\sin \theta$ tends to θ^c, and therefore $a = -g \theta = - gx/L$

where x is the distance of the bob from the midpoint of the oscillation. The acceleration is proportional to the negative of the displacement - simple harmonic motion.

The value of ω^2 is g/L, and so the period of a simple pendulum is:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

(this formula is only accurate for small angles of swing, however).

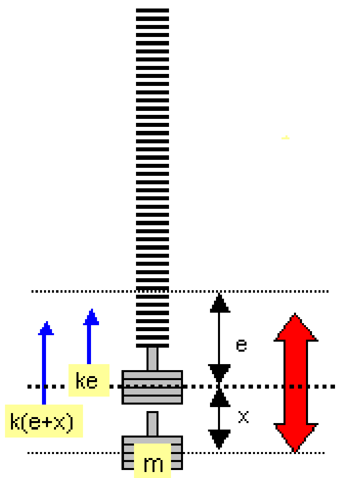
A simple pendulum may be used to measure the acceleration due to gravity (g). The period is measured for a series of different values of L and a graph plotted of T^2 against L.

The gradient of this graph is L/T^2 and this is equal to $g/4\pi^2$.

Therefore $g = 4\pi^2 L/T^2$

From this the value of g can be found. Very accurate determinations by this method have been used in geophysical prospecting.

The spring-mass system



Consider a mass m suspended at rest from a spiral spring and let the extension produced be x. If the spring constant is k we have:

mg = ke

The mass is then pulled down a small distance x and released. The mass will oscillate due to both the effect of the gravitational attraction (mg) and the varying force in the spring (k(e + x)).

At any point distance x from the midpoint:

restoring force = k(e + x) - mg

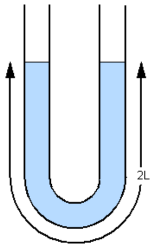
But F = ma, so ma = - kx and this shows that the acceleration is directly proportional to the displacement - SHM
The negative sign shows that the acceleration acts in the opposite direction to increasing x.

From the defining equation for SHM ($a = -\omega^2 x$) we have $\omega = k/m$
and therefore the period of the motion T is given by:

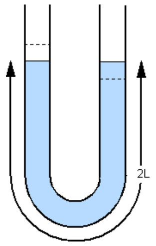
$$T = 2\pi\sqrt{\frac{m}{k}}$$

Liquid in a U-tube

Imagine a U tube containing a length 2L of liquid.



If the liquid in the U-tube is now displaced slightly and then released it will oscillate with simple harmonic motion.



The period (T) of the motion is given by the equation:

$$\boxed{\text{Period of oscillation (T)} = 2\pi[L/g]^{1/2}}$$

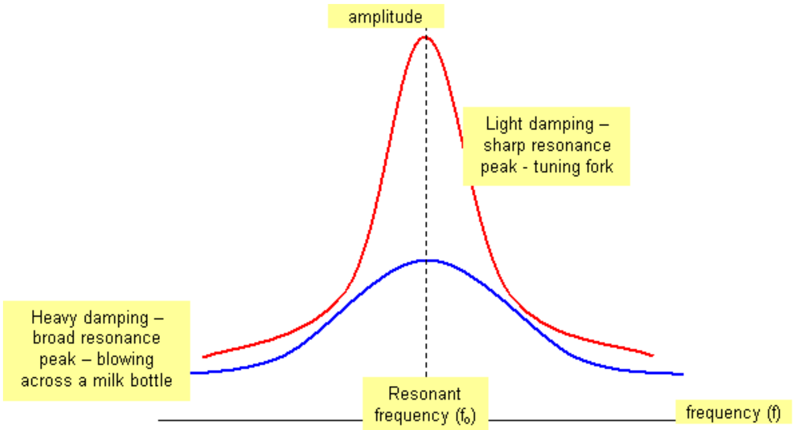
where L is half the length of the liquid in the U- tube.

Resonance

All systems have their own natural frequency, and if you apply a driving force of the same frequency and in phase with the initial oscillations then resonance results, the amplitude of the oscillations gets larger and larger. Parts of a car may vibrate if you drive over a bumpy road at a speed where the vibrations transmitted to the body are at the resonant frequency of that apart. Bass frequencies from stereo speakers can make a room resonate.

An alarming example of the effect of resonance was when the Tacoma Narrows suspension bridge collapsed in a gale, due to resonance.

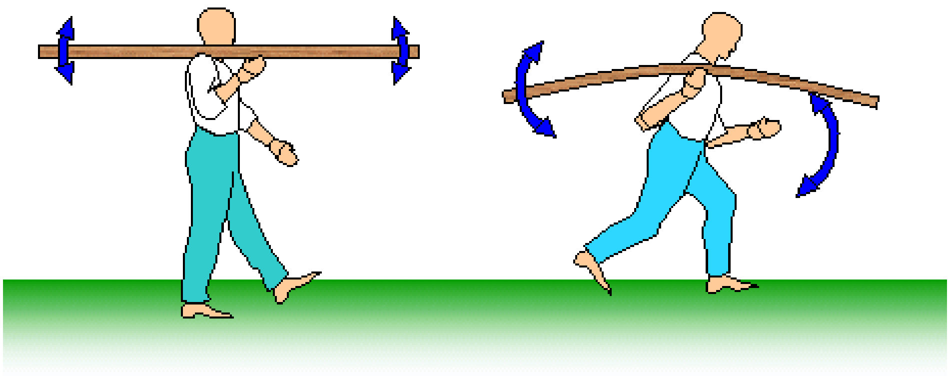
The variation of amplitude of the system with input frequency:



No mechanical system will vibrate at only its resonant frequency. The actual dependence of the amplitude of the system on the frequency of the driving force varies over a range of frequencies near the resonant frequency and this variation known as a resonance curve. The amount of damping of a system affects the shape of the resonance curve.

Heavily damped systems give broad resonance curves while lightly damped systems give sharply peaked resonance curves

A man walks across a field carrying a long plank on his shoulder.



At each step the plank flexes a little and the ends move up and down. He then starts to trot and as a result bounces up and down. At one particular speed resonance will occur between the motion of the man and the plank and the ends of the plank then oscillate with large amplitude.

Damped oscillations

Damped oscillations are oscillations where energy is taken from the system and so the amplitude decays. They may be of two types:

(i) Natural damping, examples of which are: internal forces in a spring, fluids exerting a viscous drag.

(ii) Artificial damping, examples of which are: shock absorbers in cars, interference damping - gun mountings on ships.

Energy in simple harmonic motion

The kinetic energy of any body of mass m is given by kinetic energy = ½ mv^2
In simple harmonic motion:

$$\boxed{\text{Kinetic energy} = \tfrac{1}{2} m\omega^2(r^2 - x^2)}$$

The maximum value of the kinetic energy will occur when x = 0, and this will be equal to the total energy of the body. Therefore:

$$\boxed{\text{Total energy} = \tfrac{1}{2} m\omega^2 r^2}$$

Therefore, since potential energy = total energy – kinetic energy, the potential energy at any point will be given by:

$$\boxed{\text{Potential energy} = \tfrac{1}{2} m\omega^2 x^2}$$

A graph of the variation of potential energy, kinetic energy and the total energy